

Problem Set 2

Math 257a

October 4, 2019

1. Consider $T^*\mathbb{R}^n$ with its standard symplectic structure. Symplectomorphisms of $T^*\mathbb{R}^n$ are called canonical transformations in the physics literature. Read a little bit about the generating functions approach to finding canonical transformations (Wikipedia article on Canonical transformation is good, but you might want to ignore the parts containing Hamiltonians, as we did not quite get there. Make sure to look at the type 1-4 generating functions).

Now read the paragraph “If we happen to have a symplectomorphism ...” on page 5 of Weinstein. Find 5 different ways in which a function on \mathbb{R}^{2n} can give rise to a canonical transformation of $T^*\mathbb{R}^n$ via that paragraph. Make the connection with what you read for the previous paragraph.

2. Find the article “Characteristic class entering in quantization conditions” by Vladimir Arnold. Write down full proofs of Lemma 1.1 and Lemma 1.4.1.
3. Find the article “The Sturm theorems and symplectic geometry” also by Vladimir Arnold. Write down full proofs of the two propositions in Section 3.
4. Using these two articles (or any other resource you would like to use) explain two ways in which one can assign an integer to a path $\gamma : [0, 1] \rightarrow Sp(2n, \mathbb{R})$, $n > 0$, such that $\gamma(0) = I$ and $\gamma(1)$ does not have 1 as an eigenvalue. This assignment is supposed to be invariant under continuous variations among such paths, and also take all the integers as its values for some path. You don’t need to prove your statements but the procedures should be clearly explained.