1. Assuming that Sard’s theorem for smooth maps $U \to \mathbb{R}^n$, where $U$ is an open subset of an Euclidean space, is true, prove Sard’s theorem for general manifolds.

2. Prove Sard’s theorem for maps $\mathbb{R}^3 \to \mathbb{R}^2$. You should consult your book but please do the final write-up by yourself. You do not need to write a proof of Fubini’s theorem or what they call in the book “Taylor’s theorem”.

3. Assume that $f : [0, 1] \to \mathbb{R}$ is a smooth map. Prove that there exists a $c > 0$ such that for every $x, y \in [0, 1]$ the following inequality holds:

   $f(x) < f(y) + f'(y)(x - y) + c(x - y)^2$.

4. If you had any unsolved problems or exercises from class, please do them this week. Generally go back and review the stuff we did so far. I gave you a shorter than usual homework so that you have time to do these.