

Problem Set 4

Math 257a

November 1, 2019

1. Solve Homework 17 from Ana Cannas da Silva's book - available here: <https://people.math.ethz.ch/acannas/Papers/lsg.pdf>
2. Work out moment maps for the Hamiltonian actions of \mathbb{R}^3 and $SO(3)$ on $T^*\mathbb{R}^3$, corresponding to translations and rotations of \mathbb{R}^3 . Make the connection with linear and angular momentum from classical mechanics.
3. Find the survey article "Moment maps and geometric invariant theory" by Chris Woodward, which is available on <http://sites.math.rutgers.edu/ctw/quotients.pdf>. Explain Example 3.3.3 and Proposition 3.3.4 in detail.
4. Find the paper "On the Flux Conjectures" by Lalonde-McDuff-Polterovich - <https://arxiv.org/pdf/dg-ga/9706015.pdf>. Prove the exact sequence on page 1 in detail (feel free to consult the cited resources). What is the corresponding exact sequence of "Lie algebras"?
Give a short explanation of the classical Flux conjecture using the paper. Find out if this is still a conjecture by doing a little research.
5. Prove that any Hamiltonian isotopy (i.e. a diffeotopy consisting of Hamiltonian diffeomorphisms) is generated by a time-dependent Hamiltonian vector field. You may want to wait until we cover the Weinstein neighborhood theorem for this one, but you could start by thinking why this is relevant at all. There is a clue in the first three pages of the paper from the previous problem.