

Problem Set 5

Math 257a

November 18, 2019

1. Find the paper “On global action-angle coordinates” by Duistermaat. What is being proven in Theorem 1.1? What are action-angle coordinates? Compare the statement to Weinstein neighborhood theorem for a Lagrangian torus.
2. Do Exercise 3.4.12 from McDuff-Salamon “Introduction to Symplectic Topology” 2017 Edition. Would you expect a similar phenomenon happen for the neighborhood of a Lagrangian?
3. Find the paper “On coisotropic embeddings of pre-symplectic manifolds” by Gotay. Write down and prove the normal form theorem for neighborhoods of coisotropic submanifolds. You could have done this without the paper.
4. Find the paper “Symplectic embeddings of polydisks” by Guth - <https://arxiv.org/pdf/0709.1957.pdf>. Read the proofs of Polterovich’s lemma and the Main Lemma from page 4. Sketch the one for Polterovich’s lemma.
5. If you were interested in what I called “Katok embedding” in class, you can read about them in the paper “Ergodic perturbations of degenerate Hamiltonian systems” of Katok (Main Lemma in Section 3.) Note that the triangle means “symmetric difference”. A good first step might be to understand how to use this statement to find a counterexample to Gromov non-squeezing if we only require 99.999 percent of the ball to embed into the cylinder (the rest can go anywhere in the Euclidean space, but Katok actually controls this to some extent as well).
6. Find something that interests you in the paper “Symplectic rigidity of Lagrangian submanifolds” by Audin-Lalonde-Polterovich and prepare a 10 minute presentation about it. You will be presenting in the last two lectures. This is for fun (I hope), I will not be grading you on these presentations.